数学与系统科学研究院

计算数学所系列学术报告

报告人：Dr. HOLGER HEUMANN

（ INRIA, EPI CASTOR, France ）

报告题目：

EULERIAN AND SEMI-LAGRANGIAN METHODS FOR ADVECTION-DIFFUSION FOR DIFFERENTIAL FORM

邀请人：　郑伟英 研究员

报告安排：

Lecture 1: ***Advection-diffusion problem for differential forms***

2014 年 **5** 月 **22** 日（周四）上午 **9:00-10:00** 科技综合楼 **311** 报告厅

Lecture 2: ***Yee's scheme and differential forms for Maxwell's equation***

2014 年 **5** 月 **23** 日（周五）上午 **10:00-11:00** 科技综合楼 **301** 报告厅

Lecture 3: ***Stabilized Galerkin methods and semi-Lagrangian methods for advection diffusion***

2014 年 **5** 月 **24** 日（周六）上午 **10:00-11:00** 科技综合楼 **311** 报告厅

Abstract:

Thinking in terms of co-ordinate free differential forms offers considerable benefits as regards the construction of structure preserving spatial discretizations. By now this approach is widely appreciated for boundary value problems with second order differential operators, such as curl curl, where the so-called discrete exterior calculus [1, 4, 11], or, equivalently, the mimetic finite difference approach [3, 12], or discrete Hodge-operators [2,10] have shed new light on existing discretizations and paved the way for new numerical methods. All these methods have in common that l-forms are approximated by l-co-chains on generalized triangulations of the computational domain . This preserves deRham co-homology and, thus, plenty of algebraic properties enjoyed by the solutions of the continuous boundary value problem carry over to the discrete setting. Moreover, simplicial and tensor product triangulations allow the extension of co-chains to discrete differential forms, which furnishes structure preserving finite element methods.

In light of the success of discrete differential forms, it seems worthwhile exploring their use for more general equations. In these talks we will discuss numerical methods for the advection-diffusion of differential forms. Advection-diffusion of differential forms is a generalization of the well-known scalar advection diffusion and includes important model problems for magnetohydrodynamics. As in the scalar case, the stability of spatial discretization for dominant advection is a key issue.

In the first talk I introduce basic concepts for differential forms, state the advection-diffusion problem for differential forms and present a well-posedness result for the transient problem. I show that the eddy current model in moving media can be cast into this framework. The second talk will be about the classical Yee's finite difference scheme [13] for Maxwell's equations, that serves as a first motivation for the use of discrete differential forms. Yee's scheme for Maxwell's equation will be complemented by more recent extension to magnetohydrodynamics [5]. In the third and fourth lecture I will introduce stabilized Galerkin methods and semi-Lagrangian methods for advection diffusion of differential forms [6, 7] and present recent convergence results [8, 9].

References

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