数学与系统科学研究院

计算数学所学术报告

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报告题目：

NUMERICAL METHODS TO SOLVE STOCHASTIC NONLINEAR SCHRODINGER EQUATION IN STRATONOVICH SENSE

邀请人： 陈志明 研究员

报告时间：2017 年 1 月 19 日（周四）

上午 9:00-10:00

报告地点： 数学院南楼二层

202 教室

Abstract:

In this talk, we consider numerical methods applied to the stochastic Schrodinger equation in Stratonovich sense. Based on a variational principle with a stochastic forcing, we present the general formula of the infinite-dimensional stochastic Hamiltonian system, and indicate that the stochastic Schrodinger equation in Stratonovich sense is an infinite-dimensional stochastic Hamiltonian system, whose phase ow preserves the infinite-dimensional stochastic symplecticity. It is due to the multiplicative noise in the sense of Stratonovich that the charge conservation law still holds; however, the energy is no longer preserved, and satisfies a stochastic evolution relationship. The stabilities of solution in H1(O) and H2(O) are proved.

In order to design numerical methods which preserve the infinite-dimensional stochastic symplecticity of the original problem, we propose a class of stochastic Runge-Kutta methods to discretize the temporal direction of stochastic Schrodinger equation, and obtain the symplectic conditions for stochastic Runge-Kutta methods. It is shown that under the symplectic conditions, stochastic Runge-Kutta methods preserve the discrete charge conservation law. We present a fundamental convergence theorem on the mean-square convergence orders of the semi-discretizations to stochastic Schrodinger equation, which establishes the mean-square convergence order of a method resting on its local error and properties of semigroups only. It is a crucial criterion to evaluate the mean-square convergence order. Based on this theorem, we obtain the mean-square convergence order of midpoint scheme, which verifies the theorem.

In order to design numerical methods which may inherit the stabilities of the solution, we propose a class of θ-method to discretize the temporal direction of stochastic Schrodinger equation. To generate enough numerical dissipativity to control discretization effects of the noise term, we choose θ from the range [1/2 + c √τ ; 1] with c ⩾ c* > 0. Moreover, the convergence order of this scheme is analyzed under diffierent assumptions.

At last, a damped stochastic nonlinear Schrodinger equation is studied, which is ergodic. In order to inherit numerically the ergodicity, we propose a fully discrete scheme, whose spatial direction is based on spectral Galerkin method and temporal direction is based on a modification of the implicit Euler scheme. We not only prove the unique ergodicity of the numerical solutions of both spatial semi-discretization and full discretization, but also present error estimations on invariant measures, which gives order 2 in spatial direction and order 1/2 in temporal direction under certain hypotheses.

欢迎大家参加！