The Least-squares Mixed Finite Element Methods for The Thin Plate Problem and The Incompressible Magnetohydrodynamic Problem

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Abstract

Many mathematical models in science, technology and engineering can be described as differential equations. The finite element methods are effective method to solve these equations. Least-squares mixed finite element methods are mostly used for high-order differential equations or the the differential equations which have two and more variables. The main idea of the least-squares mixed finite element methods discussed in this thesis is: transformation of the primary differential equations into first order system by using some new variables, introduced a least-squares functional and study the resulting minimization problem in the framework of the Lax-Milgram theory establishing the boundness and the coercivity in an appropriate space.

In this thesis we discuss the least-squares mixed finite element methods for the thin plate problems and the incompressible Magnetohydrodynamic(MHD) Equations in detail, and we prove the coerciveness of the variational problems corresponding to the thin plate problems and the MHD equations. Moreover, we can get the conclusion that the weak solutions are well-posed.

For MHD equations, we employ least-squares functionals which involve a discrete inner product which is related to the inner product in $H^{-1}(\Omega)$. The use of this inner product results in a method of approximation which is optimal with respect to the required regularity as well as the order of approximation even when applied to problems with low regularity solution.

Keywords: least-squares mixed finite element method, the thin plate problems, the incompressible Magnetohydrodynamic, negative norm.