Theories and Algorithms for Several Nonlinear Matrix Equations

Guo Xiao-xia

Abstract

Nonlinear matrix equations often arise in areas of scientific and engineering computing. In this thesis, we discuss symmetric nonlinear matrix equation (NME), nonsymmetric algebraic Riccati equation (NARE), and quadratic matrix equation.

For symmetric nonlinear matrix equation (NME), we study existence and uniqueness of its HPD solutions. Moreover, we reveal essential properties of these HPD solutions and the maximal one. In particular, for the maximal HPD solution , we investigate its sensitivity property in detail and, based upon this property, we derive a computable bound for its numerical approximations. Besides, we generalize all these results to a general nonlinear matrix equation.

In this thesis, we also discuss both theories and algorithms about the nonsymmetric algebraic Riccati equation(NARE). Firstly, we study perturbation bound and structured condition number about its minimal nonnegative solution, obtaining a sharp perturbation bound and an accurate condition number. Secondly, by using the matrix sign function we present a new method for finding the minimal nonnegative solution.Based on this new method, we show how to compute the desired M-matrix solution of a special quadratic matrix equation. Finally, we propose two new feasible and effective methods, i.e., alternately linearized implicit(ALI) iteration method and structure-preserving doubling algorithm(SDA), for computing the minimal nonnegative solution of the nonsymmetric algebraic Riccati equation. Under suitable conditions, we establish the convergence theories for these two methods. The theoretical analyses show that the ALI and SDA iteration matrix sequences are monotonically convergent to the minimal nonnegative solution of the NARE. The ALI method is strong nonlinearity, and the SDA method is quadratically convergent allowing the simultaneous approximations to the minimal nonnegative solutions of the NARE as well as its dual equation. Numerical experiments show that ALI method and SDA are feasible and effective, and they can outperform the Newton and the fixed-point iteration methods.

Furthermore, by employing a special matrix pair, we present an SDA for computing the extremal solution of the general quadratic matrix equation. This method has all the merits of the above SDA but monotonicity.