The study of the finite element methods for the contact problem in elasticity

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Abstract

This thesis concerns mainly about the finite element numerical approximation for the contact problem in the application of mechanics, physics and engineering. The mathematical model of the contact problem is elliptic boundary value problem which will result in the equivalent variational inequality by using the variational theory. The main research work we do in this thesis can be divided into four parts.

The first part of this thesis is to study the scalar Signorini problem. We handle this problem by considering a scalar contact model and using nonconforming Crouzeix-Raviart linear finite element to approximate it. We derive the same error accuracy as the continuous linear finite element method. The numerical experiments verify our theoretical results and show that under the same mesh triangulation, the error of Crouzeix-Raviart nonconforming approximation is better than that of continuous linear approximation.

The second part of this thesis is to consider the contact problem in elasticity. We first discuss the contact problem between a elastic body and a rigid body. We use finite element methods to approximate the mixed formula derived by the dual problem, improve the theoretical results in [**?**], and obtain optimal or quasi-optimal convergence rate. Then, we study the contact problem between two elastic bodies with matching or non-matching meshes. we approximate the mixed formula derived by relaxing the constraint in the primal problem. We use continuous $P_2 - P_1$ element to approximate the displacement field and the contact stress respectively. We make full use of the regularity of the solution, save the computational cost compared with paper [**?**] and the error accuracy can be optimal or quasi-optimal.

The third part of this thesis aims to study the plate contact problem. We provide the optimal error estimates using nonconforming finite element methods.

The last part four, we establish a post-processing method for the second order elliptic problem in mixed finite element method. Our method improves the accuracy of the solution and the proof of this method is simple and without adding extra computational cost. Numerical examples show its efficiency.

Keywords: finite element methods, variational inequality, contact problem, error estimation, numerical analysis