Applications of Direct Method to Discrete and Semi-Discrete Integrable Systems

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Abstract

In this thesis, by using Hirota's method, we mainly discuss how to seek new coupled integrable systems and high-dimensional integrable systems. Moreover, we study the integrability and algebraic structures of the discrete and semi-discrete integrable systems.

In details,

1. In the early 1990s, Hirota and Ohta developed a procedure for generalizing equations from the KP hierarchy to produce coupled systems of equations, which we now call pfaffianization. In this thesis, we successfully apply the method to semi-discrete cases and obtain coupled systems for the semi-discrete Toda equation and a special lattice. Moreover, we present the bi-directional Wronskian determinant solution to the molecule equation corresponding to the Backlund transformation of the special lattice.
2. As we know, solutions to many bilinear equations can be expressed by determinants or pfaffians. For example, the solutions to KP hierarchy can be expressed as Wronskian determinants, Casorati determinants or Gram determinants. The solutions to the coupled KP hierarchy and BKP hierarchy can be expressed by pfaffians. By using this fact, we further study Gram determinant solutions and Gram-type pfaffian solutions to the 2-dimensional Toda lattice equation, the differential-difference KP equation, the semi-discrete Toda equation and the discrete KP equation and their corresponding coupled systems. Here we would like to point out that it is much more difficult to find out solutions for the discrete KP equation and its coupled system. Moreover, we consider the multi-soliton solutions to the higher-order Ito equation.
3. Corresponding to the generalization of continuous cases, such as KP equation and NNV equation are generalizations of KdV, we present a (2+1)-dimensional strong generalization of Lotka-Volterra equation and discuss its Lax pairs, Backlund transformation and DKP-type pfaffian solutions. As an example, we obtain exact solutions of physical significance such as solitions and dromions. Furthermore, by taking the Backlund transformation as a new soliton equation which is again a strong generalization of the potential form of a self-dual nonlinear network equation, we consider its integrability. Here, we combine the compatibility of pfaffian elements and Lax pairs and define pfaffian elements by the eigenfunctions of Lax pairs.
4. Lax pairs and conservation laws are very important properties shared by integrable systems. Based on Lax pairs, we deduce an infinite number of conservation laws for high-dimensional differential-difference equations in a systematic way.

Keywords: **pfaffianization, pfaffian solutions, the symmetric (2+1)-dimensional Lotka-Volterra equation, the modified (2+1)-dimensional Lotka-Volterra equation, conservation laws**