Adaptive PML Technique for Time-harmonic Scattering Problems

Xuezhe Liu

Abstract

The Bérenger perfectly matched layer(or PML)is used in computational electromagnetism as a "sponge layer" to terminate finite element approximation of scattering problems.An infinite PML creates no reflection for incident waves of any frequency or any incident direction,and the waves decay exponentially in magnitude into the layer.This property is lost when the layer is truncated since the truncation boundary generates a reflected wave.Discretization of the differential equations further changes the reflection coefficient of the layer.

The focus of this thesis is on the Adaptive Pefectly Matched Layer Technique for Time-harmonic Scattering Problems . The PML parameters such as the thickness of the layer and the fictitious medium property are determined through sharp a posteriori error estimates.For the purpose of mathematical analysis,we exploit an uniform estimate for the Hankel functions H_ν^1, $\nu \in \mathbb{R}$,:

$$|H_\nu^{(1)}(z)| \le e^{-\mathrm{Im}\,(z)\left(1-\frac{\Theta^2}{|z|^2}\right)^{1/2}} |H_\nu^{(1)}(\Theta)|,$$

for any $z \in \mathbb{C}_{++}, \Theta \in \mathbf{R}$ such that $0 < \Theta \le |z|$, where $\mathbb{C}_{++} = \{z \in \mathbb{C} : \mathrm{Im}\,(z) \ge 0, \mathrm{Re}\,(z) \ge 0\}$. This sharp estimate, which seems first appeared in this paper.

The derived finite element a posteriori estimate for adapting meshes has the nice feature that it decays exponentially away from the boundary of the fixed domain where the PML layer is placed. This property makes the total computational costs insensitive to the thickness of the PML absorbing layers. Numerical experiments are included to illustrate the competitive behavior of the proposed adaptive method.

Keywords: Adaptivity, perfectly matched layer, a posteriori error analysis, Hankel function,scattering problems.