Finite Volume Element Method and Its cascadic multigrid Scheme

Abstract

The important and crucial property of inheriting the physical conservation laws of the original problem locally makes the finite volume element(FVE) method more attractive in the numerical computing fields. Being involved with this popular numerical method, the dissertation includes two parts. In the first part, we mainly discuss the FVE discretization scheme of the P_1-nonconforming quadrilateral element($\mathcal{NC}^h$) for elliptic problems and its application in solving the planar elasticity problem. Considering the peculiar characteristic of the element $\mathcal{NC}^h$, for the general quadrilateral partition, we define its FVE discretization scheme for elliptic problems over the overlapped dual partition and obtain optimal error estimates. Numerical experiments of different types of meshes support our theory strongly. On the other hand, existing convergent analysis and numerical experiments shows that the finite element method(FEM) of this element is locking-free when being used to solve the nearly incompressible pure displacement planar elasticity problem. In the dissertation, we obtain optimal locking-free error estimates for this discretization problem by giving error estimates of the Stokes problem first when it was discretized by the mixed element $(\mathcal{NC}^h, Q_0)$. Based on this result, we use the P_1-nonconforming quadrilateral FVE method to discretize the planar elasticity problem and obtain locking-free optimal estimates.

The second part is devoted to discuss how to solve the FVE discretization problems of elliptic equations by cascadic multigrid method. The non-symmetry of the discrete systems brings many difficulties in computing, but noting that the FVE bilinear form $a_h^*(\cdot,\cdot)$ is only a small disturbance of the FEM bilinear form $a_h(\cdot,\cdot)$, we solve a symmetric auxiliary problem about $a_h(\cdot,\cdot)$ instead of the original nonsymmetric problem. The cascadic multigrid algorithms for the P_1-conforming triangular and P_1-nonconforming quadrilateral FVE discretization problems are discussed, theoretical analysis shows that the algorithms we propose is of optimal accuracy and computational complexity. Peculiarly, for the P_1-nonconforming quadrilateral element, we define a new inter-grid transfer operator to ensure the convergence of its cascadic multigrid algorithm.

Keywords: nonconforming quadrilateral element, finite volume method, finite volume element method, cascadic multigrid method