Finite Element Method for the Inhomogeneous Dielectric Waveguide

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Waveguide problems are very important in scientific and engineering computing. Advances in various branches of photonics technologies have established the need for the development of numerical and approximate methods for the analysis of a wide range of waveguide structures that are not amenable to exact analytical studies [37]. The focus of this thesis is on the finite element analysis and computing for the electromagnetic guided waves in a closed, inhomogeneous, pillared three-dimensional waveguide at a given frequency.

We consider in this thesis a closed waveguide defined by a right cylinder with cross section Ω, a bounded, Lipschitz, simply connected polyhedral domain in $\mathbf{R}^2$. The waveguide is filled with inhomogeneous media whose electromagnetic properties are described by the real-valued functions ε and μ. The electric field and magnetic field in waveguide are given by the Maxwell Equations. The problem is formulated as a generalized eigenvalue problem. We introduce a new mixed variational eigenvalue problem to describe the electromagnetic guided waves in waveguide. The key technical difficulty is the proof of the inf-sup condition of the modified sesquilinear form which allows us to use the general framework for the approximation of the eigenvalue problems developed in [10]. We introduce a finite element method which uses the lowest order Nédélec edge element and standard conforming linear finite element to approximate the electric field components in the plane of the cross section and the electric field components along the waveguide, respectively. By modifying the sesquilinear form associated with the eigenvalue problem, we provide new convergence analysis for the finite element approximations. Numerical results of the waveguide with simple structure are reported to illustrate the performance of the finite element method. Because the eigenfunctions have singularities which deteriorate the finite element convergence, the associated numerical complexity of the waveguide with complex structure are not quasi-optimal.

Self-adaptive discretization methods have gained an enormous importance for the numerical solution of partial differential equations that arise from physical and technical applications. The aim is to obtain a numerical solution with a prescribed tolerance using a minimal amount of work. The main tool are posterior error estimators and indicators which enable us to give global and local information on the error of the numerical solution using only the computed numerical solution and known data of the problem.

By modifying the inverse iteration finite element method of eigenvalue problem, we introduce a new adaptive inverse iteration finite element algorithm. The posterior error estimators and indicators for inhomogeneous dielectric waveguide problems have been constructed in the adaptive inverse iteration finite element algorithm. Numerical results of inhomogeneous dielectric waveguide with complex structure are reported to illustrate the performance of the method.

Keywords: Dielectric Waveguide, Finite Element, Eigenvalue Problem, Inverse Iteration Method, Posterior Error Estimators, Adaptive Algorithm