The Definite Generalized Eigenvalue Problem: A Reworked Perturbation Theory

(Joint work with Roy Mathias)

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Chi-Kwong Li 是美国 **William & Mary** 学院数学系的 **Ferguson** 冠名教授, 数学系系主任. 他也是香港大学名誉教授, 国际线性代数学会期刊委员会主席, 以及国际权威杂志 **"Linear Algebra Appl.", "Linear and Multilinear Algebra" "J. Inequalities in Pure and Appl. Math."** 编委和国际线性代数学会会刊 **"IMAGE"** 的资深副主编. 获得过 **"Phi Beta Kappa Faculty Teaching Award (2003)"** 和 **"Virginia Outstanding Awards (2004)"**. **Li** 教授主要从事矩阵, 算子和组合理论方面的研究工作, 特别是在 **"Matrix Inequalities", "Numerical Ranges"** 和 **"Linear Preserver and Isometry Problems"** 等领域具有很高的造诣.

Abstract

Let (A, B) be a definite pair of $n \times n$ Hermitian matrices. That is, $|x^*Ax| + |x^*Bx| \neq 0$ for all non-zero vectors $x \in \mathbf{C}^n$. It is possible to find an $n \times n$ non-singular matrix X with unit columns such that

$$X^*(A + iB)X = \mathrm{diag}(\alpha_1 + i\beta_1, \ldots, \alpha_n + i\beta_n)$$

where α_j and β_j are real numbers. We call the pairs (α_j, β_j) *unit generalized eigenvalues* **of the definite pair (A, B). These pairs have not been studied previously. We rework the perturbation theory for the eigenvalues and eigenvectors of the definite generalized eigenvalue problem $\beta Ax = \alpha Bx$ in terms of these unit generalized eigenvalues and show that they play a crucial rule in obtaining the best possible perturbation bounds. In particular, one can replace most instances of the Crawford number**

$$c(A, B) = \min\{|x^*(A + iB)x| : x \in C^n, x^*x = 1\}$$

with the larger quantity

$$d_{\min} = \min\{|\alpha_j + i\beta_j| : j = 1, \ldots, n\}$$

in existing perturbation bounds. This results in bounds that can be stronger by an arbitrarily large factor. We also give a new measure of the separation of the jth eigenvalue from the kth:

$$|(\alpha_j + i\beta_j)\sin(\arg(\alpha_j + i\beta_j) - \arg(\alpha_k + i\beta_k))|.$$

This asymmetric measure is entirely new, and again results in bounds that can be arbitrarily stronger than the existing bounds. We show that all but one of our bounds are attainable. We also show that the Crawford number is the infimum of the distance from a definite pencil, *a fortiori* **diagonalizable, to a non-diagonalizable pair.**