Convergence properties of collocation methods for a Volterra integral equation with weakly singular kernel

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Abstract

We consider the following second kind Volterra integral equation

$$y(t) = \int_0^t \frac{s^{\mu-1}}{t^\mu} y(s)ds + g(t), \quad t \in (0, T], \tag{1}$$

where $\mu > 0$ and g is a given function (see e.g. [2]). If $0 < \mu < 1$, then (1) has a family of solutions in $C[0, T]$, of which only one has C^1 continuity. If $\mu > 1$ and g belongs to $C^m[0, T]$ then (1) has a unique solution in $C^m[0, T]$. Several papers have been devoted to the numerical treatment of (1). In the case $\mu > 1$, the construction and analysis of convergence of a class of product integration methods based on interpolatory quadratures was investigated in [2]. In [3] a Hermite-type collocation method was studied while in [4] an extrapolation method based on Euler's method was developed. Recently we have been concerned with collocation methods based on general polynomial spline spaces and the possibility of superconvergence is under investigation. Some numerical examples are shown which illustrate the performance of the methods. It should be noted that, owing to the particular nature of the kernel, the convergence arguments used for weakly singular equations of Abel type (see e.g. [1]) cannot be applied to the present equation.

References

[1] H. Brunner and P.H. Van der Houwen, *The Numerical Solution of Volterra Equations* (North-Holland, Amsterdam, 1986).

[2] T. Diogo, N.B. Franco and P. Lima, High order product integration methods for an integral equation with weakly singular kernel, *Commun. Pure Appl. Anal.* **3** (2004) 217-235.

[3] T. Diogo, S. McKee and T. Tang, A Hermite-type collocation method for the solution of an integral equation with certain weakly singular kernel, *IMA J. Num. Anal.* **11** (1991) 595-605.

[4] P. Lima and T. Diogo, An extrapolation method for a Volterra integral equation with weakly singular kernel, *Appl. Num. Math.* **24** (1997) 131-148.