UNIFIED A POSTERIORI ERROR ANALYSIS OF NONSTANDARD FINITE ELEMENT METHODS

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Totally different finite element methods are summarized under one common technique. Surprisingly, this remains one type of residuals Res for different problems, such as, the Laplace problem, the Stokes problem, and Navier-Lamé problem, with conforming, non-conforming and mixed finite element method. The main observation is that

$$\operatorname{Res}(v) := \int_\Omega g \cdot v dx + \int_{\cup \mathcal{E}} g_{\mathcal{E}} \cdot v ds \quad \text{for } v \in V$$

s the same for all those schemes. Some nonconforming elements are depicted in the following tables.

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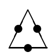
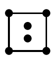
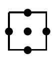
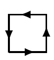
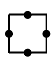
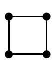
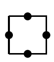
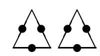
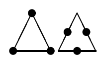
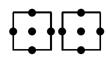
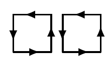
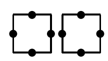
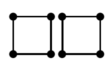
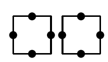
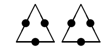
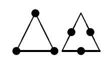
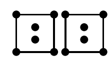
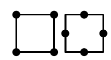
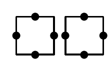
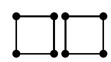
	Crouzeix-Raviart
	Wilson
	Han
	NR (M)
	NR (A)
	CNR
	DSSY

NCFEM for Laplace

picture	name
	Crouzeix-Raviart
	Kouhia-Stenberg
	Han
	NR (M)
	NR (A)
	HMS
	CJY

NCFEM for Stokes

picture	name
	Brenner-Sung
	Kouhia-Stenberg
	Zhang
	Ming
	LLS
	HMS

NCFEM for Navier-Lamé

The conclusion of this presentation is sparsity in the mathematical research of a posteriori error control. The reduction is to two parts. (a) Analyze your new PDE in such a way that the error is equivalent to $\| \operatorname{Res} \|_*$ and analyze $V_h \subset \ker \operatorname{Res}$. (b) Design new a posteriori error estimates for $\| \operatorname{Res} \|_*$.

The presentation is partly based on joint work with Jun Hu and Antinio Orlando.

References

[1] C. Carstensen, A unifying theory of a posteriori finite element error control, Numer.Math, 100 (2005),617–637.

[2] C. Carstensen, Jun Hu, A. Orlando, Framework for the a posteriori error analysis of nonconforming finite elements, Preprint (2005-11), Department of Mathematics, Humboldt University of Berlin (2005). Accepted by SINUM 2006.

[3] C. Carstensen and Jun Hu, A Unifying Theory of A Posteriori Error Control for Nonconforming Finite Element Methods, Preprint (2006-21), Department of Mathematics, Humboldt University of Berlin (2006).