数学与系统科学研究院

计算数学所学术报告

(定期学术报告)

报告人： 李杰权教授

(首都师范大学数学系)

报告题目：

The Generalized Riemann Problem(GRP) Scheme for Compressible Fluid Flows

邀请人： 洪佳林研究员

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计算数学所报告厅

Abstract：

The Generalized Riemann Problem (GRP) for a

nonlinear hyperbolic system of m balance laws (or alternatively “quasi–conservative” laws) in one space dimension is now well–known and can be formulated as follows: Given initial–data which are analytic on two sides of a discontinuity, determine the time evolution of the solution at the discontinuity. In particular, the GRP numerical scheme (second–order high resolution) is based on an analytical evaluation of the first time derivative. It turns out that this derivative depends only on the first–order spatial derivatives, hence the initial data can be taken as piecewise linear. The analytical solution is readily obtained for a single equation ($m = 1$) and, more generally, if the system is endowed with a complete (coordinate) set of Riemann invariants. In this case it can be “diagonalized” and reduced to the scalar case. However,most systems with $m > 2$ do not admit such a set of Riemann invariants. This paper introduces a generalization of this concept: weakly coupled systems (WCS). Such systems have only “partial set” of Riemann invariants, but these sets are weakly coupled in a way which enables a “diagonalized” treatment of the GRP. An important example of a WCS is the Euler system of compressible, nonisentropic fluid flow ($m = 3$). The solution of the GRP discussed here is based on a careful analysis of rarefaction waves. A “propagation of singularities” argument is applied to appropriate Riemann invariants across the rarefaction fan. It serves to “rotate” initial spatial slopes into “time derivative”. In particular, the case of a “sonic point” is incorporated easily into the general treatment. A GRP scheme based on this solution is derived. Special attention is given to the “acoustic approximation” of the analytical solution. It can be viewed as a proper linearization (different from the approach of Roe) of the nonlinear system. The resulting numerical scheme is the simplest (second–order, high–resolution) generalization of the Godunov scheme.

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