## Abstract

Let $(c_i)$ be a given sequence of numbers. We define the linear functional $c$ on the vector space of polynomials by

$$c(x^i) = c_i, \quad i = 0, 1, \ldots, \quad \text{and} \quad c(x^i) = 0, \quad i < 0.$$

The family of polynomials $\{P_k\}$ is called the family of *formal orthogonal polynomials* with respect to $c$ if, for all $k$, $P_k$ has the exact degree $k$ and

$$c(x^i P_k(x)) = 0, \quad i = 0, \ldots, k-1.$$

When $c_i = \int_a^b x^i w(x)\, dx$, where $w$ is a weight function, the usual orthogonal polynomials are recovered.

Formal orthogonal polynomials satisfy many of the algebraic properties of the usual ones: a three-term recurrence relation, the Shohat-Favard theorem, the Christoffel-Darboux identity and its consequences, etc.

Formal orthogonal polynomials play a fundamental role, for example, in the construction of Padé approximants, and in the algorithms for implementing Lanczos method for solving systems of linear equations.