Abstract

Let f be a formal power series

$$f(z) = c_0 + c_1 z + c_2 z^2 + \cdots$$

We are looking for a rational function

$$R(z) = \frac{a_0 + a_1 z + \cdots + a_p z^p}{b_0 + b_1 z + \cdots + b_q z^q}$$

such that its power series expansion in ascending powers of z (obtained by division) agrees with that of f *as far as possible*, that is

$$f(z) - R(z) = O(z^{p+q+1}). \tag{1}$$

Such a rational fraction is called a *Padé approximant* of f.

Of course, the main purpose of Padé approximants is to approximate functions given by a formal series expansion. In this respect they achieved great successes and they were much useful in solving problems in theoretical physics, fluid dynamics, numerical analysis, ...

Padé approximants have also many other applications in the solution of various questions of numerical analysis not so closely related to power series approximation. In particular, Padé approximants and their generalizations have applications in the construction of A-acceptable approximations to the exponential function, a question of primary importance in the numerical solution of differential equations. They have also been applied to the Laplace transform inversion and to the Borel transform. By their link with the ε–algorithm, they have applications in the solution of systems of nonlinear equations where they provide a method with quadratic convergence (under certain assumptions) which does not require the computation of derivatives. In the case of a system of linear equations, they are related to Lanczos method and to other projection methods for solving systems of linear equations. They also have applications in the computation of the eigenvalues of a matrix, and the treatment of the Gibbs phenomenon for Fourier series..