Convergence analysis for numerical methods to stochastic hyperbolic equations

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Abstract

For a simple model of a scalar wave equation with random wave speed, Gottlieb and Xiu (Commun. Comput. Phys., 3 (2008), pp. 505-518) employed the generalized polynomial chaos (gPC) method and demonstrated that when uncertainty causes the change of characteristic directions, the resulting deterministic system of equations is a symmetric hyperbolic system with both positive and negative eigenvalues. Consequently, a consistent method of imposing the boundary conditions is proposed and its convergence is established under the assumption that the expansion coefficients decay fast asymptotically. In this work, we investigates stochastic collocation methods for the same type of scalar wave equation with random wave speed. It will be demonstrated that the rate of convergence depends on regularity of the solutions, which is determined by the random wave speed and the initial and boundary data. Numerical examples are presented to support the analysis and also to show the sharpness of the assumptions on the random wave speed and the initial and boundary data.