数学与系统科学研究院

计算数学所系列学术报告

报告人： **Prof. Wei-Chi YANG**

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报告题目： **Series of Seminars on Integrating Technological Tools into Solving Optimization Problem**

邀请人： 林群 院士

报告安排：

Lecture 1:2013 年 9 月 3 日（周二）上午 9:30-11:00，科技综合楼 311

Lecture 2:2013 年 9 月 5 日（周四）上午 9:30-11:00，科技综合楼 311

Lecture 3:2013 年 9 月 10 日（周二）上午 9:30-11:00，科技综合楼 301

Lecture 4:2013 年 9 月 12 日（周四）上午 9:30-11:00，科技综合楼 311

Lecture 5:2013 年 9 月 17 日（周二）上午 9:30-11:00，科技综合楼 311

Lecture 6:2013 年 9 月 19 日（周四）上午 9:30-11:00，科技综合楼 311

Lecture 7:2013 年 9 月 24 日（周二）上午 9:30-11:00，科技综合楼 311

Lecture 8:2013 年 9 月 26 日（周四）上午 9:30-11:00，科技综合楼 311

欢迎大家参加！

Series of Seminars on Integrating Technological Tools into Solving Optimization Problems

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Abstract

The proposed series of eight seminars (60 minutes lecture and 30 minutes discussion) is to demonstrate how technology has prompted us to rethink the way we research and discover mathematics. Graphical and geometrical representations are crucial for making mathematical conjectures before utilizing a computer algebra system in assisting us of proving or disproving our conjectures analytically. The lectures are most derived from publications at eJMT (http://ejmt.mathandtech.org). Many of which were inspired by technological tools. Particiapnts will gain knowledge on how mathematics and its applications can be linked through using the latest technological tools. Participants will gain appreciation of how mathematics can be made fun and yet challenging. General areas of discussion will include optimization problems in 2-D and 3-D.

Speaker: Wei-Chi Yang, Professor of Mathematics and Statistics at Radford University, Virginia. Founder: Asian Technology Conference in Mathematics (*ATCM: atcm.mathandtech.org*). Founder and Editor-in-Chief: Electronic Journal of Mathematics and Technology (*eJMT:ejmt.mathandtech.org*) and the Research Journal of Mathematics and Technology (RJMT: rjmt.mathandtech.org)

1 Selected Topics

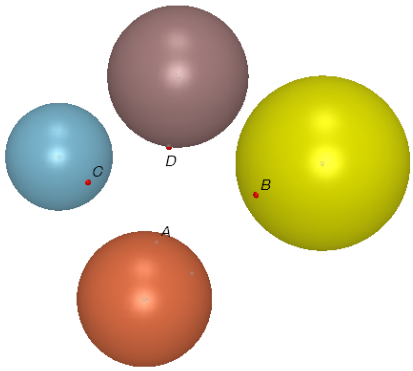
Lectures 1 and 2. *Caustic curves and surfaces.* We first discuss finding the *inverse image* of a parametric curve with respect to $y = mx + b$, and we extend the idea to three dimensions. Next, we find the reflection of a light source on a curve C_1 with respect to a moving point P on C_2, which we call it P'. We find the **locus of the point** P', which links to the concept of **orthotomic and caustic curves.** The concepts can be extended to the corresponding concepts in 3-D. It is interesting to see how technological tools have prompted us to

relate mathematics to similar concepts used in physics (see [3]). In particular, *we shall discuss why the 3D caustic surface is still a challenging problem for many researchers.*

When light reflects off a convex curved surface, the reflected rays form a curve of bright light called a caustic. This curve is exactly the envelope curve of the reflected rays. Figure 12 shows the caustic curve formed by light reflecting in a wedding ring.

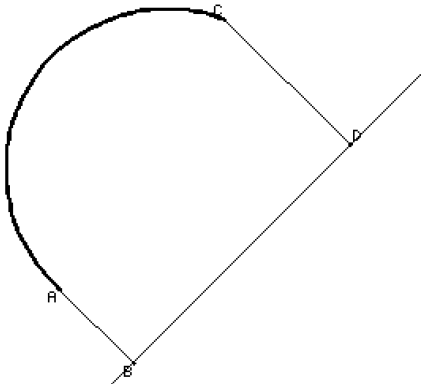
Figure 12: Caustic curves formed by light reflecting inside a gold ring

Lecture 3. *An Optimization Problem.* We will explore some examples in finding the global values of a *total squared distances* among non-intersecting curves in the plane and surfaces in the space. The geometric interpretations will help students appreciate the use of the Lagrange multipliers method and concepts learned from linear algebra. Some generalizations will be discussed. For example, given the following four surfaces (orange, yellow, blue and purple), we would like to find the minimum of the total square distances: $AB+AC+AD$ (see [2]). *We shall discuss how the methods used in this case can be extended to solving more complex real-life problems.*



Lecture 4. *Finding signed Areas and Volumes.* We describe how we can find the *signed area* bounded by a parametric curve with respect to a slanted line, $y = mx+b$, in two dimensions. We extend this idea to the corresponding setting in three dimensions. These are special cases of the Green's and the Divergence

Theorems, but the ideas and methods introduced here are accessible to a wider audience (see [4]). *We also describe how to find the volume bounded by a surface and a slanted plane in 3D case.*



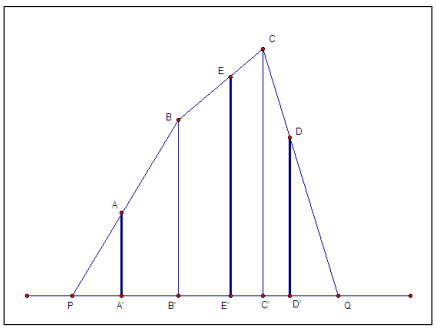
Lecture 5. *Geometric proofs for Mean Vlaue, Cauchy Mean Value Theorems and Higher dimensions of Mean Value Theorems.* We will demonstrate geometric insights and motivations of the proofs of the Mean Value and *Cauchy mean value* theorems (see [5]).

Suppose the function $f : [a, b] \to R$ and $g : [a, b] \to R$ are continuous and that their restrictions to (a, b) are differentiable. Moreover, assume that $g'(x) \neq 0$ for all x in (a, b). Then there is a point t in (a, b) at which

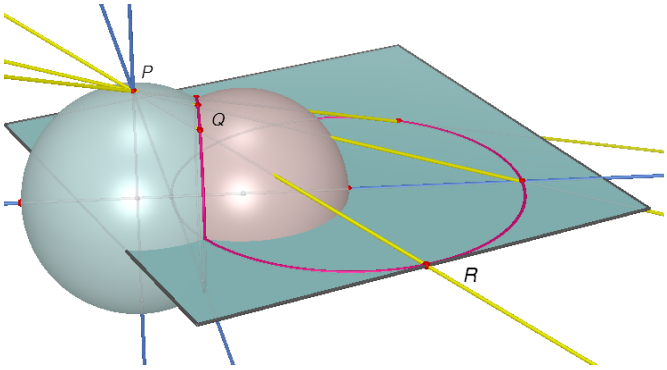
$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(t)}{g'(t).}$$

We will describe how the 3D Mean Value Theorem can be stated and how it is related to optimization problems with constraints and Lagrange multipliers methods can be used (see [6]).

Lecture 6. *Another Optimization Problem (Three Ladders-Walls Problem)*: We are given a set of numerical data for the heights of three respective ladders and the widths between two ladders. For example, we let $h_1 = \frac{165}{100}, h_2 = \frac{225}{100}, h_0 = \frac{295}{100}, d_1 = -\frac{205}{100}$ and $d_2 = \frac{70}{100}$. We shall minimize the total length of $PBCQ$. We shall discuss how this problem is applied in real-life problems (see [7]).

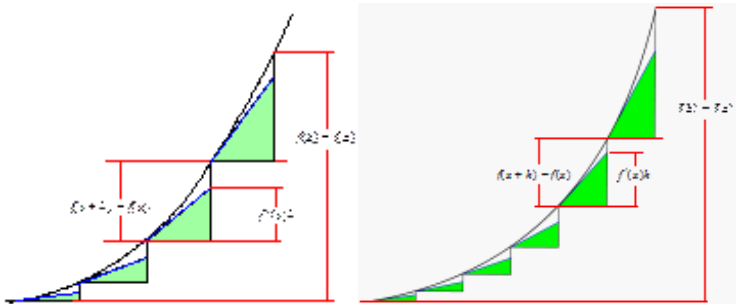


Lecture 7. *Shrinking* Circle and *Sphere:* We are given two spheres that are intersecting at a circle. We fix the sphere on the right (pink), consider the projection by connecting the north pole of the sphere on the left (point P) and every point on the intersecting circle. We want to find the locus of such projection when the radius of the sphere on the left is shrinked to 0, see [7]). We shall discuss how the 3D problem can be extended to higher dimensions and how the problem is related to Meusnier Theorem.



Lecture 8. *Extension of the Fundamental Theorem of Calculus (FTC): The description of FTC can be simply understood by the following visualization (see [1]). We will describe how the FTC can be extended to the Green Theorem for line integral in higher dimensions. We shall discuss how a partition can be done unevenly to speed up the convergence or divergence, and to handle improper*

Riemann Integrals.



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2. Yang, W.-C., 'Some Geometric Interpretations of The Total Distances Among Curves and Surfaces', the Electronic Journal of Mathematics and Technology (eJMT), ISSN 1933-2823, Issue 1, Volume 3, 2009.

3. Yang, W.-C. & Lo, M.-L., , 'General Inverses in 2-D, 3-D, applications inspired by Technology', the Electronic Journal of Mathematics and Technology (eJMT), ISSN 1933-2823, Issue 2, Volume. 2, 2008.

4. Yang, W.-C. & Lo, M.-L.,'Finding signed Areas and Volumes inspired by Technology', the Electronic Journal of Mathematics and Technology (eJMT), ISSN 1933-2823, Issue 2, Volume. 2, 2008.

5. Yang, W.-C., 'Revisit Mean Value, Cauchy Mean Value and Lagrange Remainder Theorems', the Electronic Journal of Mathematics and Technology (eJMT), ISSN 1933-2823, Issue 2, Volume. 1, 2007.

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8. Meade, D. & Yang, W.-C., 'Analytic, Geometric, and Numeric Analysis of the Shrinking Circle and Sphere Problems', the Electronic Journal of Mathematics and Technology (eJMT), ISSN 1933-2823, Issue 1, Volume 1, 2007.