数学与系统科学研究院

计算数学所学术报告

报告人： **Prof. Rong Tian**

（ ***Institute of Computing Technology, CAS*** ）

报告题目：

Extra-dof-free and linearly independent enrichments in GFEM/XFEM

邀请人： 赵旭鹰 博士

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计算数学所小报告厅

Abstract:

The core of a generalized finite element method (GFEM) is a Partition of Unity (PU) approximation (in 1D):

$$u^h(x) = \sum_{i \in I} N_i u_i + \sum_{i \in I} N_i \sum_k \varphi_k^{P_i} a_{i(k)}$$

where $\sum_{i=1}^{N} N_i(x) \equiv 1$ form the PU, $\varphi_k^{P_i}$ is the user-defined local function on patch P_i which is composed of elements surrounding node *i*, and $a_{i(1)}$, $a_{i(2)}$, ... are the extra dofs of node *i*.

The GFEM offers tremendous advantages over the classical finite element approximation by its user-tailorable enrichment and its excellent accuracy and convergence properties. Nevertheless, there are still critical issues that render difficulties in practical engineering.

One issue is the extra nodal dof. The number of dofs per node in the existing GFEM counts 9*p*(*p*+1)/2+3 in 3D for a complete polynomial local approximation of order *p*—the number fast increases with *p*. The extra dof results in a fast expanding linear system for a high order of local approximation. In addition, the extra dof to a certain extent hinders the progress of the GFEM in dynamic problems because the mass matrix associated with the method remains open. The other known issue is the so-called linear dependence: when both the PU and the local function are polynomial, the global stiffness matrix is singular.

A GFEM without extra dof has been developed. The new GFEM inherits the excellent accuracy and convergence properties of the existing GFEM and additionally offers excellent stability. The new method has no extra nodal dof, is free of linear dependence and is significantly good conditioned. The excellent stability and the extra-dof-free feature would be greatly rewarded in large-scale problems and applications to dynamic analyses (the mass matrix now can be defined in a usual way).

The new GFEM interpolates as long as the local approximation at least interpolates at the patch star, no matter that the local approximation is in nature an interpolation or an approximation. The RBF and the Selectively Interpolating MLS/LS have been developed to construct the extra-dof-free local approximation on an unstructured mesh of any dimension, without losing the Kronecker delta property.

A simple approach is proposed to recover linear consistency on the blending elements in the adaptive implementation. Numerical tests show the excellent performance of the approach—the adaptive GFEM offers a clean and smooth transition from the FEM to the GFEM approximation without complicating implementation.

In summary, although much remains to be investigated, we believe that the potential of the new GFEM developed herein is very attractive.

The paper has just been published in Computer Methods in Applied Mechanics & Engineering (one of top-tier journals in computational mechanics communities). One of reviewers gave very positive comments on this piece of work: "*The author presents a new type of approximation in the family of partition of unity methods and GFEM. I would like to congratulate the author to this excellent piece of research work. In the number of different formulations that have developed over the last 10 years (XFEM, GFUM, PUM, intrinsic XFEM/PUM, hp-clouds etc.) this work succeeds in suggesting yet another, innovative yet not far-fetched or excessively complicated general formulation of an approximation. Therefore, I very much recommend the publication of this work...*"

References:

[1] Babuška I, Melenk JM. Partition of unity method. *International Journal for Numerical Methods in Engineering* 1997; 40:727 –758.

[2] Rong Tian. Extra-dof-free and linearly independent enrichments in GFEM. *Computer Methods in Applied Mechanics & Engineering* 2013; 266: 1-22

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